Using Digital Fabrication to Support Student Learning

Kimberly Corum[1] and Joe Garofalo[1]

Abstract

Desktop digital fabrication technologies provide students with access to concrete and virtual manipulatives, which have both been identified as useful instructional tools to support student learning in a variety of different content areas, such as mathematics. In particular, these technologies can be used to help support students' development of conceptual understandings of three-dimensional measurement. This article describes how a digital fabrication-augmented unit supported the teaching and learning of surface area. Our goal was to see how working with both virtual and concrete manipulatives affected students' development of strategies to use when solving surface area tasks. Fifth-grade students used modeling software and die cutters to print physical models (three-dimensional cubes and rectangular prisms) from digital designs, giving them access to virtual and physical manipulatives. There was substantial pretest–posttest improvement on students' performance on surface area tasks following their participation in the digital fabrication-augmented unit. Additionally, features of the software and the unit supported students' development of two strategies: (1) being aware of nonobservable faces of prisms in two-dimensional representations of three-dimensional figures and (2) keeping track of their work.

Introduction and Background

Desktop digital fabrication technologies, such as desktop 3D printers and digital die cutters, are being employed in a variety of contexts. Digital manufacturing has been used in industry since the 1950s. However, with these machines becoming smaller and more affordable, digital manufacturing is becoming a technology that is more accessible for individual consumers. As is the case with many developing technologies, the question of how these technologies can be incorporated into the classroom to support student learning is one that continues to be explored. Schools across the country are purchasing die cutters and 3D printers and these technologies are being used in conjunction with modeling software to create both virtual and physical models. These models are used in a variety of contexts, such as after-school clubs or technical courses. However, digital fabrication can be utilized in regular classroom settings as well. 3D printers, die cutters, and related software provide students with access to concrete and virtual manipulatives, which have both been identified as useful instructional tools to support student learning in a variety of different content areas, such as mathematics.

Digital Fabrication and Learning

Concrete manipulatives are physical objects that are used in the classroom to support student learning. Two separate meta-analyses of research on the effectiveness of concrete manipulatives in mathematics instruction conclude that the use of manipulatives resulted in improved student achievement.[1,2] However, these two analyses did not identify the conditions under which the use of manipulatives during classroom instruction is most effective.

A virtual manipulative is defined as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge."[3] A study of the effects of virtual manipulatives on fifth-grade students' learning of surface area and volume found that "Interaction among multiple representations, including manipulation of the 3-D shapes and literacy on the white board, encourages students to interpret mathematical meanings from different viewpoints."[4] Digital fabrication provides students with the ability to interact with both concrete manipulatives and virtual

[1]Mathematics Education, Curry School of Education, University of Virginia, Charlottesville, Virginia.

MARY ANN LIEBERT, INC. • VOL. 2 NO. 2 • 2015 • DOI: 10.1089/3dp.2015.0008

manipulatives, which allow students to access multiple representations.

The benefit of students working with multiple representations is consistent with a number of past and current learning frameworks. Tall and Vinner discuss learning in terms of students' *concept images,* where a concept image refers to "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes."[5] Here mental pictures include "any kind of representation—picture, symbolic form, diagram, graph, etc."[6] Mayer's theory of multimedia learning states that learning involves building connections among pieces of verbal knowledge to create a coherent verbal model and also building connections among chunks of pictorial knowledge to create a coherent pictorial model. According to Mayer, a crucial step "involves a change from having two separate representations—a pictorial model and a verbal model—to having an integrated model in which corresponding elements and relations from one model are mapped onto the other."[7] By providing students with both virtual and concrete manipulatives, digital fabrication can be used to help students connect mental representations to construct meaningful coherent concept images.

Learning Surface Area

Digital fabrication can be used to help support students' development of conceptual understandings of three-dimensional measurement. In particular, this technology can be used when teaching surface area and volume. Both the National Council of Teachers of Mathematics and the Common Core State Standards indicate that students from upper elementary grades through 12th grade should understand surface area and volume.[8,9] However, National Assessment of Educational Progress results indicate that students struggle with two-dimensional measurement tasks, such as measuring length and calculating area and perimeter.[10] Typical classroom instruction that overemphasizes the use of formulas or introduces formulas prematurely can impede students' conceptual understandings and lead to conceptual misundertandings.[10,11] These findings extend to measuring surface area and volume.

The literature led us to believe that there may be a benefit to using concrete and virtual manipulatives when teaching surface area. In this brief article, we report on an early project exploring the degree to which a simple die cutter and modeling software in a digital fabrication-augmented surface area unit affected fifth-grade students' ability to solve surface area tasks. The data for this article were collected in 2011, when additive manufacturing was not as accessible in schools. By using computer-aided design (CAD) software and die cutters to construct physical models of rectangular prisms, students also had virtual versions of the prisms and their corresponding nets to support their learning.

Materials and Methods

Digital fabrication is defined as, "the process of translating a digital design developed on a computer into a physical object."[12] Teachers can use the computers in their classrooms to create personal fabrication systems by installing CAD software and adding 3D printers or die cutters.[12,13]

Digital fabrication was used to support the teaching and learning of surface area and volume. Two instructional units (one focusing on surface area and another focusing on volume) were developed that incorporated *FabLab ModelMaker* (Aspex Software) and *Silhouette* die cutters during classroom instruction. This technology provided students with the opportunity to print physical models (three-dimensional cubes and rectangular prisms) from digital designs, giving students access to virtual and physical manipulatives. The digital fabrication-augmented units were taught in a fifth-grade classroom at an elementary school located in central Virginia, not far from a state university. Participating students' ability to solve surface area tasks was assessed with a project-made pretest and a posttest consisting of open-ended tasks. The posttest was given at the conclusion of both units (see the Results section for specific task examples). This article focuses on students' pretest and posttest performance and strategies on the surface area tasks.

The units consisted of a premade sequence of lessons, which loosely guided classroom instruction. The lessons included teacher-led discussions and demonstrations of the digital fabrication software, as well as a series of student hands-on tasks, during which students used the software to construct cubes and rectangular prisms. The surface area unit took place during five class periods (ranging from 30 to 50 minutes per period) over the course of a month. For a fuller description of the study, see Corum and Garofalo.[14]

Features of FabLab ModelMaker

ModelMaker allows students to design 3D models by combining geometric shapes. Using the software, students are able to rotate, transform, translate, and measure different attributes of their model. Figure 1 is a screenshot from the program; user tools are accessible along the top, bottom, and left-hand edge of the screen. This screenshot shows the optional split-screen net view of the 3D model.

The net includes tabs that allow students to more easily connect the different faces of their model. Students can print the model's corresponding net on cardstock, vinyl, or other types of material and then use a die cutter to cut the other edges of the model and perforate the fold lines to make construction of the physical model easier.

How Our Students Used ModelMaker

Students first began their exploration of surface area by looking at different plastic models of three-dimensional shapes and identifying key attributes. By the second day of instruction, students were using the ModelMaker software to create their own 1-inch cardstock cubes (Fig. 2). This allowed students to interact with both virtual and physical models. On the third day, students were asked to define surface area; the accuracy of their definitions improved when they had access to their physical cubes. Students then used ModelMaker to measure different parameters of their 1-inch

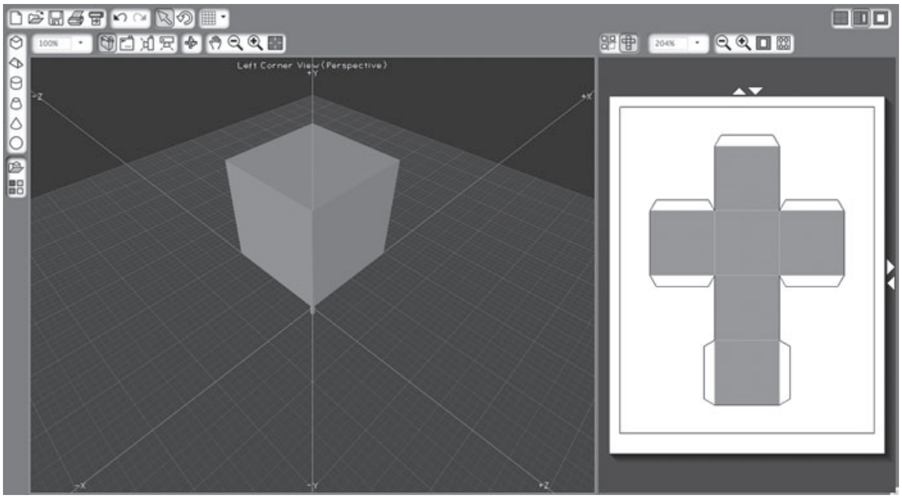

Figure 1. Screenshot of a rectangular prism designed in ModelMaker.

Figure 2. Student using the ModelMaker software and Silhouette die cutter.

cubes, such as the area and perimeter of each face.

On the fourth day, students explored the relationship between their folded cubes and their nets. Using ModelMaker, the teacher showed students how the cube could be rotated to expose nonvisible faces and edges and how the cube could be unfolded into the net view. This led to students hypothesizing different possible formulas for surface area. Students then used ModelMaker to create a 1-inch-by-2-inch-by-3-inch prism to test their hypothesized formulas. Several students were inspired to ask about the surface area of other solids, such as pyramids, cones, and spheres. On the final day, the teacher reviewed the definition of surface area and showed students how they could use ModelMaker to rotate their prisms and color individual faces. The assigned face colors were also visible on the corresponding net. Students then used ModelMaker themselves to practice rotating prisms, coloring faces, and keeping running totals of face areas.

Assessing Student Understanding

Our goal was to see how working with both virtual and concrete manipulatives affected students' development of strategies to use when solving surface area tasks. Students were assessed with a project-designed pretest and posttest that aligned with and extended beyond both state and national content standards for surface area and volume. A total of four surface area tasks were analyzed for this article. These tasks included three open-ended tasks, asked on both the pretest and the posttest, as well as an item from a Virginia Department of Education seventh-grade mathematics standardized end of course exam, which was asked only on the posttest. Inclusion of the standardized item provided an opportunity to compare our students' performance with statewide student performance.

The open-ended tasks were scored using a rubric that emphasized the conceptual knowledge required for solving surface area tasks (see Table 1). Two doctoral students in mathematics education, one of whom is the first author, scored the tasks independently. Initial interscorer agreement was 94%, and after discussing their scores, the two scorers agreed on all point allocations. Neither scorer was involved with the classroom instruction nor the data collection. Students' written work was analyzed to identify students' potential problem-solving strategies and how digital fabrication supported the development of these strategies.

Results

Students' pretest and posttest performance on the three open-ended surface area tasks is summarized in Table 2. On the pretest, not one student earned a full score on any of the open-ended tasks. In contrast, 19 students, 19 students, and 16 students earned at least 5 points on Tasks 1, 2, and 3, respectively, on the posttest. A score of at least five means that the students' only errors were either forgetting units or making a minor computation error.

There were several common *incorrect* strategies that students used on the pretest. These incorrect strategies included (1) calculating volume instead of surface area, (2) calculating the area of only one of the six faces, and (3) doubling, quadrupling, or squaring

Table 1. Rubric for grading surface area problems

Category	Point value	Description
Recognition	3	Recognized the need to find the area of the six faces in order to determine the surface area.
Setup	1	Set up appropriate calculations needed to correctly determine the surface area.
Computations	1	Correctly carried appropriate computations.
Units	1	Used appropriate units.

Table 2. Mean scores on surface area tasks

	Task 1	Task 2	Task 3
Pretest	0.04	0	0
Posttest	4.3	4.2	3.6

the dimensions and then finding the sum (see Fig. 3). It appears that many students tried to synthesize formulas for area and perimeter in an attempt to calculate surface area.

When analyzing student work on the posttest, two common *correct* strategies emerged. These common correct strategies included (1) computing the sum of the areas of all six faces (visible and not visible) and (2) doubling the areas of the visible faces. Figure 4 is an example of the first correct strategy and Figure 5 is an example of the second correct strategy.

In addition to the three tasks above, the posttest also included a surface area task taken from a previously administered Virginia Department of Education Standards of Learning Grade 7 Mathematics Assessment. Students were asked to determine the minimum amount of paper needed to wrap a rectangular prism. The task was presented as a multiple-choice question and possible answers included distracter choices (i.e., the volume of the rectangular prism and the sum of the area of only the visible faces). Eighteen students (out of 28) answered this question correctly, resulting in a 64% success rate. While not statistically significant, this success rate compares favorably to the statewide 53% success rate for seventh-grade students prepped for the test. Students employed the same common strategies in correctly solving this task as they did with the other surface area tasks. Students' track keeping strategies included listing the areas of all six faces (Fig. 6) or doubling the area of the three visible faces. For a fuller description of our results, including task-by-task analyses, see Corum and Garofalo.[14]

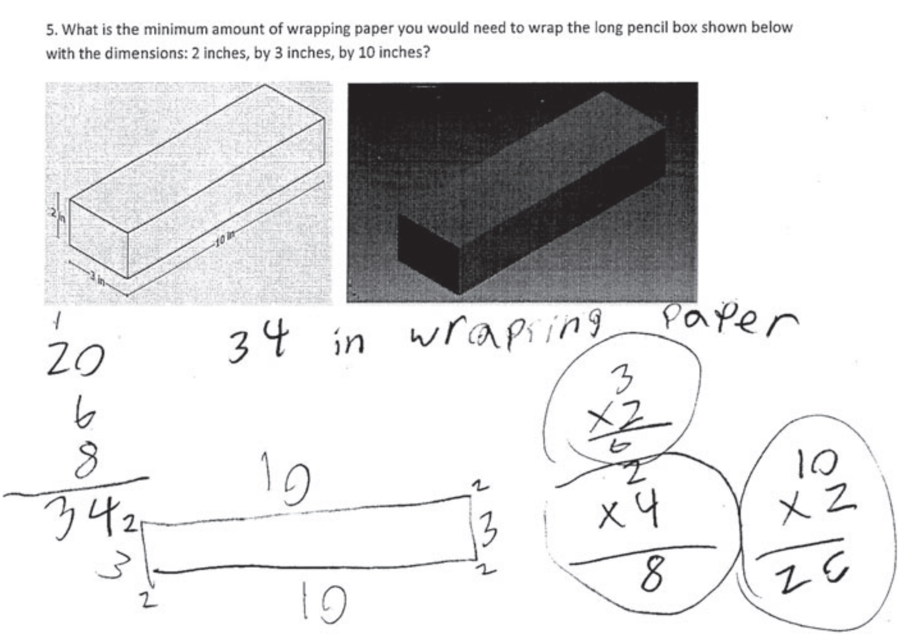

Figure 3. Pretest incorrect solution for Task 2—example of synthesizing area and perimeter formulas.

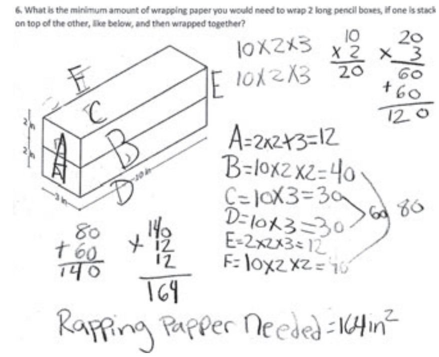

Figure 4. Posttest solution for Task 3—listing areas of each face.

Discussion

Students' performance on the surface area tasks improved dramatically after participating in the digital fabrication-augmented unit. Not one of the 28 students was able to correctly solve any of the three surface area tasks on the pretest, whereas 19 students, 19 students, and 16 students earned full or nearly full credit on Tasks 1, 2, and 3, respectively, on the posttest. By participating in the unit, students had opportunities to develop strategies that enabled them to be more successful with completing surface area tasks on the posttest. These strategies included (1) being aware of qualities of three-dimensional figures not visible in two-dimensional representations and (2) effectively carrying out multistep processes.

Student Strategies Developed

When viewing a two-dimensional representation of a rectangular prism, students are only able to see the top face, front face, and one side face. Students need to be able to visualize faces not

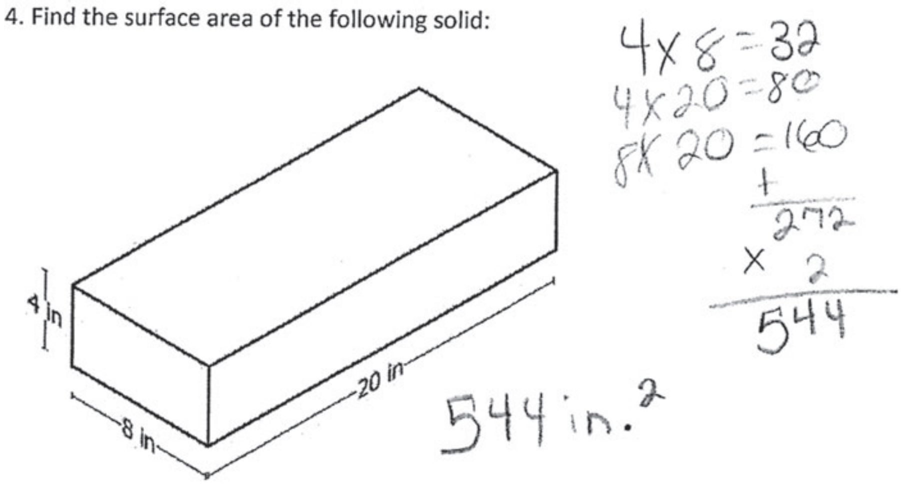

Figure 5. Posttest solution for Task 1—doubling areas of visible faces.

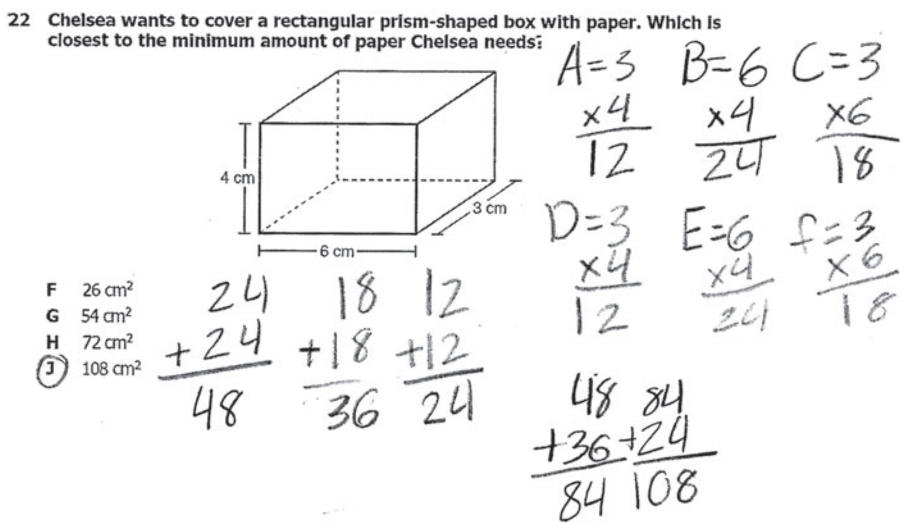

Figure 6. Posttest solution for SOL Task–listing areas of each face.

visible in a diagram in order to find the surface area. When finding the surface area of a rectangular prism, students need to recognize that each visible face has a corresponding nonvisible face. Students who successfully completed the surface area tasks on the posttest either listed the areas they calculated for each of the six faces (both visible and nonvisible) or listed the areas of the three visible faces and then doubled their areas. Both of these strategies indicate that students were *aware of the faces that were not visible in the diagram*. Students first addressed what could be seen in the diagram and then accounted for what could not be seen.

Finding the surface area of a rectangular prism is a multistep process. In addition to accounting for what was not visible in the two-dimensional representations of rectangular prisms, students who successfully completed the surface area tasks on the posttest also used strategies to *keep track of their work*. Some students *listed* areas, choosing to either list the area of each of the six faces (e.g., Fig. 6) or list the areas of only the three visible faces (e.g., Fig. 5). Some students also *labeled* or *annotated* the diagram with either the calculated area or with letters to ensure that they accounted for each face (e.g., Fig. 4).

Unit Features Facilitating Strategy Development

Several features of the unit facilitated *students' ability to consider nonvisible faces*. First, the software was used to rotate two-dimensional representations of prisms to make the initially invisible faces visible. Second, the software displayed two-dimensional representations of solids next to their corresponding nets that showed all of the faces (see Fig. 1). Finally, during the unit students physically explored their three-dimensional fabricated prisms daily.

Some aspects of the unit may have facilitated *students' keeping track strategies*. Students used the ModelMaker software to rotate virtual representations of rectangular prisms and color opposite faces, by clicking on each of the faces and choosing a color. The teacher also encouraged students to keep track when she demonstrated use of the software. Finally, students also kept track of faces when holding physical prisms in several ways: some students used their fingers as calipers to hold and count opposite faces in pairs, and some labeled counted faces with a mark, letter, or area value.

For a more thorough analysis of our results, as well as evidence of students' ability to apply both their *seeing what's not visible* and *keeping track* strategies to other tasks, see Corum and Garofalo.[14]

Conclusions

Our results show that the task performance of students who participated in the digital fabrication-augmented unit improved substantially after completion of the unit. We surmise that features of the software and the unit influenced students to consider the nonobservable faces of prisms in their calculations and keep track of their work led to this improvement. Because we did not originally conceptualize this effort as a formal study, we did not employ a control group, nor did we conduct task-based interviews of students. These omissions clearly limit our interpretation of the results. However, we are encouraged by these dramatic results and we will look at the effect of fabrication-augmented units on students' learning of other topics in mathematics and science in more comprehensive ways.

Acknowledgments

This work was partially supported by the National Science Foundation, Grant No. 1030865—The FabLab Classroom:

Preparing Students for the Next Industrial Revolution. We would also like to recognize Irina Timchenko for her work as a second scorer of student work on the surface area tasks.

Author Disclosure Statement

No competing financial interests exist.

References

1. Sowell E. Effects of manipulative materials in mathematics instruction. J Res Math Educ 1989;20:498–505.
2. Carbonneau K, Marley S, Selig J. A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. J Educ Psychol 2013;105:380–400.
3. Moyer P, Bolyard J, Spikell M. What are virtual manipulatives? Teach Children Math 2002;8:372–377.
4. Hwang W, Hu S. Analysis of peer learning behaviors using multiple representations in virtual reality and their impacts on geometry problem solving. Comput Educ 2013;62:308–319.
5. Tall D, Vinner S. Concept image and concept definition in mathematics with particular reference to limits and continuity. Educ Stud Math 1981;12:151–169.
6. Vinner S, Dreyfus T. Images and definitions for the concept of function. J Res Math Educ 1989;20:356–366.
7. Mayer R. Multimedia learning. Cambridge University Press, New York, NY, 2009.
8. National Council of Teachers of Mathematics. Principles and standards for school mathematics, volume 1. NCTM, Reston, VA, 2002.
9. Common Core State Standards Initiative. High school: geometry, geometric measurement & dimension. www.corestandards.org/Math/Content/HSG/GMD (last accessed on April 27, 2015).
10. Strutchens M, Martin W, Kenney P. What students know about measurement: perspectives from the National Assessment of Educational Progress. In: Clements D, Bright G (eds). Learning and teaching measurement: 2003 yearbook. NCTM, Reston, VA, 2003; pp.195–207.
11. Zacharos K. Prevailing educational practices for area measurement and students' failure in measuring areas. J Math Behav 2006;25:224–239.
12. Berry R, Bull G, Browning C, et al. Preliminary considerations regarding use of digital fabrication to incorporate engineering design principles in elementary mathematics education. Contemp Iss Technol Teach Educ 2010;10:167–172.
13. Bull G, Garofalo J. Personal fabrication systems: from bits to atoms. Learn Lead Technol 2009;36:10–12.
14. Corum K, Garofalo J. Learning about surface area through a digital fabrication-augmented unit. J Comput Math Sci Teach (in press).

Address correspondence to:
Kimberly Corum
Mathematics Education
Curry School of Education
University of Virginia
P.O. Box 400273
Charlottesville, VA 22904

E-mail: kimcorum@virginia.edu