MATHEMATICS IN INFORMAL LEARNING ENVIRONMENTS: A SUMMARY OF THE LITERATURE

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Updated March 2017

Research on mathematical reasoning and learning has long been a central part of the classroom and formal education literature (e.g., National Research Council, 2001, 2005). However, much less attention has been paid to how children and adults engage with and learn about math outside of school, including everyday settings and designed informal learning environments, such as interactive math exhibits in science centers. With the growing recognition of the importance of informal STEM education (National Research Council, 2009, 2015), researchers, educators, and policymakers are paying more attention to how these experiences might support mathematical thinking and learning and contribute to the broader goal of ensuring healthy, sustainable, economically vibrant communities in this increasingly STEM-rich world.

To support these efforts, we conducted a literature review of research on mathematical thinking and learning outside the classroom, in everyday settings and designed informal learning environments. This work was part of the NSF-funded *Math in the Making* project, led by TERC and the Institute for Learning Innovation and designed to advance researchers' and educators' understanding of how to highlight and enhance the mathematics in making experiences.[1] Recognizing that the successful integration of mathematics and making requires an understanding of how individuals engage with math in these informal learning environments, we gathered and synthesized the informal mathematics education literature, with the hope that findings would support the *Math in the Making* project and inform the work of mathematics researchers and educators more broadly.

Although this was not a formal synthesis, we collected literature systematically, with a focus primarily on studies since 2000. As appropriate, we reviewed seminal studies prior to this time period, such as Nunes and colleagues' groundbreaking work on everyday mathematics (Nunes, Schliemann, & Carraher, 1993; Nunes et al., 1993). Sources were identified through conversations with math education experts and systematic literature searches using PsycInfo, ERIC, Google Scholar, and informascience.org. Because there was a particular lack of research on mathematics in designed informal learning environments, we also drew from the "grey literature" in this area, including summative evaluations of museum programs and exhibits. We did not systematically review literature from the fields of adult math learning and education, although this research also offers insights into the nature of mathematics outside of school (e.g., Schmitt & Safford-Ramus, 2000; Seabright & Seabright, 2008). After reviewing identified studies, the *Math in the Making* team drafted themes and discussed these with the project advisory committee, which included experts in making, tinkering, and informal math learning.

Below we summarize findings from the review, beginning with research on everyday mathematics and followed by research and evaluation studies on math learning and thinking in designed informal learning environments. We conclude with a summary of key themes and a call to action in the hopes that this work will motivate ongoing research to understand and support how adults and children learn about and engage with mathematics outside the classroom and the important role these experiences can play in lifelong STEM learning.

[1] http://www.informalscience.org/cimble-conference-integrating-math-informal-building-learning-environments-math-making

Everyday Mathematics

Although mathematics in the classroom has received the most attention from researchers, an emerging body of literature over the last several decades makes clear that mathematical learning and reasoning are not unique to this setting. Based on research outside of school, it is clear that children and adults regularly engage with mathematics in their everyday lives and that the nature of this engagement is distinct from classroom practices. Independent of school, mathematics is a central aspect of how children and adults solve challenges and complete tasks in their everyday and professional lives (e.g., Goldman & Booker, 2009; Nunes & Bryant, 2010; Roth, 2011). Furthermore, researchers have argued that these informal experiences represent critical resources and supports for mathematics learning in formal education settings. For example, Martin and colleagues highlighted the importance of explicitly connecting in-school and out-of-school mathematics: “we believe that when the mathematics of school and that of everyday life are seen as incommensurable, it impoverishes both contexts, separating the symbolic precision and power of school math from the flexibility and creative sense-making of everyday life” (Martin & Gourley-Delaney, 2014, p. 611).

Researchers have documented mathematics and math learning in a range of everyday settings, including candy selling, carpet laying, video games, entertainment and play, sports, budgeting and money management, fishing, construction work, shopping and purchasing, farming, sewing, professional work in a variety of industries, and everyday family activities (Civil, 2002; Eloff, Maree, & Miller, 2006; Esmonde et al., 2013; Goldman & Booker, 2009; Hoyles, Noss, & Pozzi, 2001; Kliman, 2006; Martin, Goldman, & Jiménez, 2009; Martin & Gourley-Delaney, 2014; Masingila, Davidenko, & Prus-Wisniowska, 1996; Nasir, 2000; Nunes & Bryant, 2010; Nunes et al., 1993; Roth, 2011; Saxe, 1991; Taylor, 2009) For example, Nunes, Schliemann, and Carraher (1993) found that adult construction workers and fishermen who had no formal school mathematics training were able to solve proportional reasoning problems quite successfully, even compared to students who had studied proportions in school (Nunes & Bryant, 2010). Similarly, Nasir (2000) documented how high school basketball players were adept at solving basketball math problems, especially when they were allowed to use informal estimation strategies.

Mathematical reasoning and learning have also been documented as a frequent part of family experiences and parent-child interactions (Benigno, 2012; Ginsburg, 2008; Hojnoski, Columba, & Polignano, 2014; Ramani, Rowe, Eason, & Leech, 2015), including cooking, meals, chores, shopping, and play activities, and the quantity and quality of math-related experiences between parents and preschool children have been found to be important predictors of children’s developing math skills and knowledge (Ramani et al., 2015). Studying the everyday mathematical experiences of four-year-old African-American children and their families through naturalistic observation, Benigno (2012) found substantial evidence of spontaneous mathematical experiences and practices that “reflected their unique family lives, individual predispositions, and knowledge development” (p. 359), including numbers and counting, geometric thinking and spatial reasoning, and discussions of difference and similarity. The process of parents helping their children with homework, although connected with formal schooling, can also create opportunities for rich, collaborative learning for both children and adults (e.g., Ginsburg, 2008).

Unique Strategies and Goals

Studies outside the classroom have highlighted consistent distinctions between school and everyday mathematics. Research suggests that individuals are often highly pragmatic when engaging with mathematics outside of school, drawing flexibly from different strategies and resources and evaluating success based on the activity goals and outcomes, rather than the "correctness" of the answer or procedure (Hoyles et al., 2001; Martin & Gourley-Delaney, 2014; Swanson & Williams, 2014). Not surprisingly, individuals appear to primarily engage in mathematics as a way to solve specific everyday problems (Esmonde et al., 2013; Goldman & Booker, 2009; Martin et al., 2009; Masingila et al., 1996; Pea & Martin, 2010), although mathematics can also be part of entertainment or socializing

(Esmonde et al., 2013). In these situations, the problem context determines the resources and tools individuals draw on, how success is evaluated, and the salience of the mathematics (Goldman & Booker, 2009; Martin & Gourley-Delaney, 2014; Nasir, 2000; Pea & Martin, 2010; Roth, 2011; Swanson & Williams, 2014; Taylor, 2009). Compared to school math, "the approaches people take to the problems emergent for them in their practices are not constrained by school algorithms [but instead] exploit contextual features of the material and social environments and flexibly integrate the pursuit of non-math goals, such as minimizing effort or time" (Pea & Martin, 2010, p. 4). For example, Nasir (2000) found that high school basketball players performed better than non-players on basketball math problems, but only when estimation was allowed, and that they were able to use specialized problem-solving strategies they likely developed through their frequent use of player statistics. Similarly, research on nursing (Hoyles et al., 2001) indicates that while many nurses may struggle to solve abstract or decontextualized proportional reasoning problems, they regularly use proportional reasoning in their work to successfully calculate drug dosages, drawing on a variety of flexible strategies and their familiarity with specific drug units and quantities. In many cases, groups and communities have developed and evolved unique approaches to using mathematics that are suited to their specific needs, even when the underlying mathematics and mathematical problems may appear quite similar from the outside (Roth, 2011).

Particularly in the context of family interactions, actively negotiated family values and goals, such as the importance of social relationships or the minimization of time and effort, often guide how adults and children engage in mathematics (Civil & Bernier, 2006; Goldman & Booker, 2009; Pea & Martin, 2010). In these contexts, the goals of supporting socializing and social relationships may be of equal importance to individual achievement (Goldman & Booker, 2009; Kliman, 2006; Mokros, 2006). More broadly, as Martin and colleagues (2009) argued, mathematics in everyday settings is often "in the service of, and intimately tied up with, cultural goals and values. Likewise, cultural means are employed to accomplish mathematical ends" (p. 251). In the study of four-year-old African-American children cited above (Benigno, 2012), the researcher found that mathematical events "tended to: (a) emerge and evolve spontaneously from the children's intrinsic motivation, (b) demonstrate the children's meaningful application of mathematical content or active engagement in mathematical thinking as they pursued everyday goal-directed activities or engaged in mathematical meanings for its own sake, and (c) promote purpose-oriented verbal interactions (dialogue, negotiation, description) involving mathematical content between the children and significant others" (pp. 359-360). The unique goals and characteristics of specific activities and contexts, as well as broader beliefs about learning, childhood development, mathematics, and more, have all been seen to influence the nature and extent of mathematical talk and practices in families (Guberman, 2004; Ramani et al., 2015).

The types of mathematical strategies and approaches used in everyday settings also depend on the tools and resources available and the degree to which they afford, constrain, or make explicit different aspects of the mathematics (Roth, 2011; Swanson & Williams, 2014). For example, in their investigations of the mathematics used by dart players, Swanson and Williams (2014) found that mathematical aspects of play were often integrated in the artifacts of the game, including "outs tables" used to guide end-game strategies. Similarly, observing mathematical practices across four professional and school settings (a fish hatchery, a biology research laboratory, a think-aloud study of graphing expertise, and an undergraduate mathematics course on differential equations), Roth (2011) observed professionals using very different mathematical practices and strategies, even though the mathematical problems and underlying mathematics were often quite similar. The mathematics, mathematical tools, and mathematical representations often had very different meanings and functions within the different contexts and activities.

Despite its informal nature, the unique characteristics of everyday mathematics may offer distinct advantages over more traditional classroom approaches, allowing individuals to be highly accurate and successful in using math to solve everyday problems and flexible in switching approaches as needed. As noted above, in everyday settings individuals are often able to successfully answer mathematical questions, such as proportional reasoning problems, by relying on intuitive understandings of quantities and correspondence; drawing on contextual cues from the situation; using tools and manipulatives to scaffold reasoning and avoid abstract notation; using empirical approaches to develop understandings of relationships among quantities; and referring to quantities (e.g., number of fish) explicitly in their verbal reasoning, rather than only numbers or abstract ratios (Hoyles et al., 2001; Nunes & Bryant, 2010). The situated and flexible nature of everyday mathematics, as well as the possibility of using "social and empirical rules… alongside logical relationships," often makes this mathematics more accurate and foolproof than school-based math (Swanson & Williams, 2014, p. 195). For example, Fisch and colleagues (2009) observed that third and fourth grade students playing an online game shifted approaches and used increasingly sophisticated math strategies to solve game challenges when previous, simpler strategies were not effective. Drawing from Gee's theoretical work on learning through electronic games (Gee, 2007), they speculated that the informal nature of the game affords these changes by allowing for risk-taking without consequences and by creating new game scenarios and challenges that force players to "undo their routinized strategy to adapt to the new or changed conditions" (Fisch et al., 2009, p. 4).

Narrow Definitions and Perspectives

Despite the unique and often sophisticated ways that people use mathematics in their daily lives, research indicates that children and adults often have a relatively narrow perspective on what counts as mathematics and may not connect concepts or skills learned in school with their everyday mathematical reasoning (Civil & Andrade, 2002; Ginsburg, Manly, & Schmitt, 2006; Goldman & Booker, 2009; Hoyles et al., 2001; Kliman, 2006; Kliman, Jaumot-Pascual, & Martin, 2013; Masingila et al., 1996). As Kliman and colleagues (2013) noted, "even as awareness of science as a cultural and social activity is growing, adults of all backgrounds often view mathematics as a context-free topic consisting of facts and algorithms" (p. 10). Prior research in schools suggests that students tend to view mathematics as largely computational and involving problems that can be solved quickly. Students also often have difficulty finding applications for mathematics outside of school and bringing real-world knowledge to their mathematical problem-solving in the classroom (Martin & Gourley-Delaney, 2014). Outside of school, children seem to primarily associate mathematics with money, counting, and measuring, even though researchers have documented a diversity of examples of mathematical concepts and skills embedded in daily activities (Goldman & Booker, 2009; Hyatt, 2013; Jay & Xolocotzin, 2014), such as daily economics, trading and spending, counting, measuring and estimating distance and weight, exploring patterns and probability, and more. Some research suggests that even individuals in very technical fields, such as a fish culturist or field biologist, may not see themselves as doing mathematics (Roth, 2011).

A few researchers have explored and speculated about factors influencing how adults and children perceive mathematics outside of school. One study suggested that students are sensitive to the status of an activity when determining whether or not it is mathematical (Abreu & Cline, 2003). For example, a white-collar job, such as managing an office, might be more likely to be viewed as mathematical compared to a blue-collar job, such as taxi driving. Martin and colleagues (2014) found several factors that affected whether or not sixth grade students classified images of everyday and in-school activities as mathematical, including surface features, such as numbers, symbols, and money, and the possibility or necessity of mathematical action in the situation. The researchers also found that "consistent with common sense expectation, activities like dancing, playing music, and fishing were generally not seen as mathematical, while worksheets, school math presentations, and paying bills were" (p. 611). Students were also more likely to rate activities as mathematical if they had personal experience with them.

More broadly, Swanson and Williams (2014) have argued that the structure of everyday contexts, such as work environments, and the tools that we use in these situations can obscure the underlying mathematics of tasks and problems. Drawing from Vygotsky's work (Vygotsky, 1978), the researchers noted that mathematics can become "fossilized" in tools and procedures: "This fossilization (Vygotsky, 1997, p. 71) of the mathematics—often in physical artefacts, or in procedures, or fused in situated concepts—means that the acting subject is generally barely aware of the mathematics embedded there. It is concrete but not theoretical for them" (Swanson & Williams, 2014, p. 195). For example, in their research, professional and amateur dart players used "outs tables" to guide end-game strategies, based on the probabilities of achieving different combinations of points to win the game. Although these strategies are highly mathematical, "much of this know-how has been crystallised in the outs table that players can download from the internet and carry in their pockets" (Swanson & Williams, 2014, p. 198). Swanson and Williams also argued that the hierarchy and division of labor in workplaces often produces knowledge barriers that relegate the mathematical aspects of work to certain individuals and obscure or routinize the math for many other workers. This hidden nature of mathematics can break down, however, in certain situations, such as intrinsic or vocational motivation or transitions to highly competitive situations, in which individuals or groups are motivated to explore and understand the mathematics at a deeper level.

It is also worth noting that there are ongoing debates even among educators and mathematicians about the nature of mathematics and what counts as math in different settings (Martin & Gourley-Delaney, 2014; Wright & Parkes, 2015). Given this, it may not be surprising that those who do not study mathematics or math education are also confused. One helpful framework for defining mathematics in out-of-school environments has emerged from researchers studying adult education and learning, who have coined the term "numeracy" to distinguish between more formal conceptions of mathematics and those math-related topics, skills, and dispositions "woven into the context of work, community, and personal life" (Ginsburg et al., 2006, p. 1).

Social Mediation

Studies have also found that social mediation is frequently a central aspect of everyday mathematics. In the context of families, parents and caregivers often play an important role in facilitating their children's engagement with mathematics using a variety of strategies, including modeling, prompting and encouraging, engaging in distributed problem solving, asking questions, explaining and directing, or playing (Civil & Bernier, 2006; Civil, Díez-Palomar, Menéndez, & Acosta-Iriqui, 2008; Eloff et al., 2006; Goldman & Booker, 2009; Mokros, 2006). Some studies suggest that parents' cultural backgrounds and prior experiences with mathematics and school can be important influences on their approach to math learning and discourse within the family (Civil & Bernier, 2006; Guberman, 2004; Rogoff, Paradise, Arauz, Correa-Chávez, & Angelillo, 2003). Parents and caregivers often report not feeling confident in their knowledge and abilities related to helping their children learn mathematics (Lopez & Donovan, 2009; Mokros, 2006), although this may be more true in the context of math homework and school learning.

One way that parents engage children in math is through authentic involvement in everyday, mathematical activities. In studying four-year-old African-American children, Benigno (2012) documented a range of "child driven, child-and-other-driven, and adult-driven" mathematical experiences in the children's everyday lives and found that parents and other adults often played an important role by involving children meaningfully in everyday family practices through which mathematics naturally emerged, supporting mathematical understanding and exploration initiated by children, or purposely introducing and instructing children on specific mathematical skills and concepts. As the authors noted, "in child-and-other-driven mathematical events, a child's development of mathematical understandings appeared to emerge spontaneously as the child actively engaged in intent participation in a community (i.e., family) practice (Rogoff, 2003) with a significant other" (p. 366).

Family mathematics can also arise in more pedagogical contexts. For example, a small but growing body of research suggests that parent-child shared book reading experiences are important contexts of early childhood math learning (Hojnoski et al., 2014): "children's literature can be used to support early math development. Specifically, storybook text and illustrations contextualize mathematical concepts (e.g., numbers and operations, measurement, shapes), storybook reading elicits mathematical behavior (e.g., reasoning, problem solving), and the social nature inherent in shared reading mediates engagement in mathematical discourse (e.g., the parent explains or elaborates upon mathematical ideas presented by his or her child)" (p. 471).

Outside the family context, Taylor (2009) studied the mathematics of children's purchasing practices in convenience stores and found that store clerks often provided support to help children select items and make payments, especially during more complex transactions. Similarly, Nasir (2000) documented how social interactions with other players and coaches were important factors influencing the mathematical practices of middle school and high school basketball players. In the context of work settings, apprenticeship can be a common model through which adults learn and engage with mathematics (Masingila et al., 1996).

Mathematics in Designed Informal Learning Environments

Designed informal learning environments (National Research Council, 2009), such as math-themed exhibits in museums, are another setting in which rich mathematical thinking and reasoning outside the classroom can occur. Unlike schools, these settings offer individuals and groups the opportunity to more freely choose how, what, where, and with whom they learn (Falk & Dierking, 2000, 2013). However, unlike everyday settings, designed informal learning environments are often created with explicit pedagogical goals, including supporting mathematical reasoning and learning (National Research Council, 2009). Because of this, designed informal learning environments may offer rich math learning opportunities for families and children that are not widely available in formal classroom settings, including kinesthetic and social math experiences (Cooper, 2011; Wright & Parkes, 2015).

An important example of these settings is the growing number of math-focused exhibitions in museums and science centers (Cooper, 2011). Mathematics is a topic of growing interest in the informal science education field (Mokros, 2006) and there are an increasing number of museum and science center exhibitions focused on the topic, such as *Design Zone*,[2] *Math Moves*,[3] and *Geometry Playground* (Danctep, Gutwill, & Sindorf, 2015),[4] as well as a new museum focused entirely on mathematics.[5] Although they have been the focus of less research attention, libraries can also be spaces for facilitated and unfacilitated math learning experiences (e.g., Kliman et al., 2013). Similarly, online games are another opportunity for rich, informal math learning. For example, studying third and fourth graders using an online math game developed to complement the Cyberchase TV series, Fisch and colleagues (2009) observed and tracked children using a range of sophisticated mathematical strategies that often became more advanced as they played the game and encountered new scenarios and challenges.

Although the literature is small (Anderson & Thompson, 2001; Cooper, 2011), there is a growing body of research and evaluation studies providing evidence of the mathematical thinking and learning that is possible in these settings. Investigators in science centers, for example, have documented evidence of algebraic and proportional reasoning (Garibay Group, 2013a; Pattison, 2011; Pattison,

[2] http://www.omsi.edu/exhibits/designzone/
[3] http://www.mathmoves.org/
[4] http://www.exploratorium.edu/geometryplayground/
[5] http://momath.org/

Ewing, & Frey, 2012; Rubin, Garibay, & Pattison, 2016; Selinda Research Associates, 2016); spatial reasoning (Danctep et al., 2015); qualitative, intuitive understandings of slope (Nemirovsky & Gyllenhaal, 2006; Wright & Parkes, 2015); connections with the mathematics in the experiences to school and everyday lives (Garibay Group, 2013a); and more general math-related discourse, such as description, counting and numbers, patterns, size estimation, problem-solving, comparison, spatial orientation, precision, shape identification, and fractions (Randi Korn & Associates, 2001; Vandermaas-Peeler, Massey, & Kendall, 2015). However, other studies have documented lost opportunities. For example, in observations of visitors at a zoo, a children's museum, and a history museum, Cooper (2011) found abundant opportunities for mathematical learning but limited evidence of math-related conversations within families. In one of the few projects that took advantage of mathematical possibilities in institutions with live animal collections, the *Math in Zoos and Aquariums* project (Garibay, Martin, Rubin, & Wright, 2012) used animal behavior and animal characteristics as the basis for several family-oriented math activities. One challenge for the field is that the majority of evaluation studies have focused on assessing project-specific goals and outcomes (e.g., Garibay Group, 2013a, 2013b, Randi Korn & Associates, 1999, 2001), providing few details on the nature of visitor mathematical reasoning, behaviors, or conversations.

A consistent finding from studies, also aligned with research from everyday settings, is that visitors are often not aware that they are engaging with mathematics or have relatively narrow conceptions of math (Garibay Group, 2008; Gyllenhaal, 2006; Randi Korn & Associates, 1999). For example, in the front-end evaluation for the *Design Zone* project, Garibay Group (2008) noted that "both children and adults most commonly associated math with numbers and operations" (p. 4) and that even older children and adults had a limited notion of algebra beyond solving for an unknown. In the evaluation of the *Handling Calculus* exhibition (Gyllenhaal, 2006), most visitors without formal calculus experience associated the exhibit activities with math in general, rather than the specific topic of calculus. However, for those who had taken calculus courses, the experience was often connected with both positive and negative school memories. Gyllenhall (2006) also reported that some individuals can become anxious when they learn that an experience involves math, potentially because of negative previous experiences with the topic.

Given these potential negative associations, some educators and developers working in informal learning environments have attempted to address these challenges and promote awareness of the mathematics without undermining other experience and learning outcomes. For example, exhibit developers often come face to face with the need to balance these two goals when they name an exhibition. While the developers of *Geometry Playground* purposely used a math term in the title, the developers of *Design Zone* consciously avoided this association. Nonetheless, in the summative evaluation of the *Design Zone* exhibition (Garibay Group, 2013a), the majority of visitors felt that the exhibit experiences were connected to math in school or in their everyday lives. Furthermore, 95% of respondents enjoyed their experience in the exhibition and 94% of the children in the target age range (10 to 14) who remembered using math in the exhibition indicated that they felt comfortable with that aspect of the experience.

Also similar to mathematics in everyday settings, evaluation and research studies have repeatedly highlighted the importance of social mediation when visitors engage with math in designed informal learning environments. In several evaluation studies of math exhibitions at science centers and children's museums, Randi Korn & Associates (1999, 2001) found that parents and caregivers played an important role in facilitating mathematical reasoning and engagement and that the level and nature of that facilitation appeared to differ across activities. In one study, parent facilitation strategies included asking questions, making suggestions, pointing out details, instructing children, and engaging in dramatic play (Randi Korn & Associates, 1999). Similarly, in the evaluation of the *Handling Calculus* exhibition, Gyllenhaal (2006) found that adults and parents often facilitated learning for visitor groups, even when they knew little about the math content. In the summative evaluation of the *Design Zone* exhibition (Garibay Group, 2013a), evaluators found that parents and other adults played an important role in facilitating math learning and increased the likelihood that family members engaged in more sophisticated algebraic reasoning, such as conversations about the relationships between different variables in the exhibits. Aligned with the flexible nature of math outside of school, one way adults might play an important role in these interactions is by helping their groups to adopt different mathematical strategies appropriate to the level of understanding within the group and to the problem or challenge relevant at a given moment (Rubin et al., 2016).

Only a few studies have explored the design characteristics of these settings that might support, or hinder, mathematical engagement and learning. One strand of this work has focused on the influence of exhibit size and scale, and in particular differences between immersive and tabletop exhibits. For example, Dancstep and colleagues (Danctep et al., 2015) used an experimental design to compare visitor experiences and outcomes at tabletop- and immersive-versions of exhibits as part of a larger exhibition designed to support spatial reasoning. At both versions of the exhibits, adults and children used spatial language and reasoning during the interactions, including "static, dynamic, and causal" (p. 412) language. Counter to their expectations, however, the visitors at the tabletop versions exhibited higher levels of spatial reasoning language compared to the visitors at the immersive versions, on average. In contrast, building on the notion of embodied cognition in mathematics (Abrahamson & Lindgren, 2014; Eisenberg, 2009; Hall & Nemirovsky, 2012), Nemirovsky and colleagues conducted several studies demonstrating the potential of interactive and immersive exhibits for supporting visitors in the development of more intuitive understandings of mathematical relationships and concepts (Nemirovsky & Gyllenhaal, 2006; Nemirovsky, Kelton, & Rhodehamel, 2013; Wright & Parkes, 2015). Similarly, a summative evaluation of the *Math Moves!* exhibition indicated that visitor engagement "demonstrating increasing qualitative and kinesthetic fluency" (Selinda Research Associates, 2016, p. 69) was particularly noticeable at whole-body exhibits, although engagement times were longer at some smaller tabletop activities.

Another strand of research in this area has focused on supporting the role of parents or adult family members during interactions at math exhibits. Vandermaas-Peeler and colleagues (2015) found that providing parents and family groups with additional orientation and guidance by a staff member before entering a math exhibition was associated with family groups asking a greater variety of guiding questions and talking more about measurements and size comparisons. Similarly, in research and evaluation studies of the *Design Zone* exhibition, investigators found evidence that carefully designed "parent panels" with supporting information for adult family members were important for encouraging algebraic reasoning (Garibay Group, 2013a; Rubin et al., 2016). Research on *Design Zone* also highlighted the promise of clear and explicit challenges posed in exhibit labels for enhancing math exploration (Garibay Group, 2013a), as well as the potential trade-offs of using technology, such as exhibit-embedded computer guides, to prompt challenges and structure the visitor experience (Pattison et al., 2012). Emerging evidence also suggests that museum educators can enhance visitor satisfaction and mathematical reasoning at interactive exhibits when staff are supported by research-based professional development (Pattison et al., 2016, 2017).

These few studies provide early indications that, like classrooms and everyday settings, designed informal learning environments can offer rich opportunities for supporting mathematical reasoning and learning. However, with characteristics that are similar to and different from both everyday settings and classrooms, designed informal learning environments may also offer unique constraints and affordances (Rubin et al., 2016). For example, while educators and designers can provide rich mathematical representations for learners in these environments, the use of these tools may be dependent on the goals and social context of the experience. Similarly, although science centers and other informal learning environments create excellent opportunities for socially mediated learning (Astor-Jack, Whaley, Dierking, Perry, & Garibay, 2007; National Research Council, 2009; Pattison & Dierking, 2013), those in the position to support learning during these experiences, such as parents or staff facilitators, may have limited understanding of mathematical reasoning or the strategies for fostering math learning. And in contrast to classrooms, where argumentation and proof may be explicit goals (National Research Council, 2005; Yackel & Cobb, 1996), in museums and science centers such modes of discourse may be at odds with social expectations (National Research Council, 2009). Given the importance of mathematical reasoning for success in school and life (National Research Council, 2005), there is a critical need to explore these tradeoffs and investigate the potential of designed informal learning environments for supporting mathematical learning.

Conclusions

Research over the last several decades makes clear that adults and children regularly engage with and learn about mathematics outside the classroom and that these experiences offer rich opportunities for further math learning. Based on our review of the literature, we suggest several themes highlighted by existing research:

1) There is strong evidence that children and adults regularly engage in mathematical thinking and learning outside of school, in both everyday and professional settings, as well as designed informal learning environments, such as interactive exhibits.

2) The goals that individuals pursue and the strategies they use when engaging in mathematical thinking and learning outside of school are often distinct from classroom mathematics. These experiences are frequently framed by non-mathematical goals, such as efficiency or socializing, and adults and children are often quite successful at flexibly using a variety of strategies and resources appropriate to the context to accomplish these goals.

3) Mathematical thinking and learning outside of school is often done in a social context in which social mediation and facilitation, such as guidance by a parent or caregiver in a family setting, are an important aspect of the experience.

4) Despite the ubiquity of everyday mathematics, many adults and children appear to have a relatively narrow view of what counts as math and frequently do not associate their everyday mathematical practices with school math, or math at all.

What is still poorly understood is how educators and policymakers can use these findings to effectively support mathematical engagement and learning outside of school, and the role these experiences might play relative to classroom mathematics and lifelong STEM learning. Designed informal learning environments, such as interactive math exhibits in science centers, appear to be promising settings for supporting and encouraging mathematical thinking and leveraging both the unique nature of everyday mathematics and the power of purposefully designed learning tools. Additional research will help the field clarify the breadth of mathematical topics that can be productively explored in such settings and how both experiences and professional development can be designed to enhance learning outcomes and promote positive math attitudes.

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This material is based upon work supported by the National Science Foundation under Grant No. 1514726.
Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.